Announcements: HW#1: • Either drop off at Math 113 by 3pm
OR email to ksankar@math.ubc.ca by 2pm

• Solutions posted today

HW #2: Will be up today

Office Hours: LSK300 2-3PM.

Definition: Let a and b are integers. We say "a divides b" if there is an integer c s.t. $ac=b$.

$a|b$

s.t. = such that

Ex: n is even $\equiv$ 2 divides n

$2|n$

"n is a multiple of 3" $\equiv$ $3|n$

⋮

n is odd $\equiv$ 2 does not divide n

$2\nmid n$

Proposition: Let n be an integer. Then n^2-5n+2 is even.

Proof: Any integer n is either even or odd. Let's do each case separately.

Case 1: n is odd.

Case 1

Therefore n^2-5n+2 is even.

Case 2: n is even. Then there is some $a \in \mathbb{Z}$ s.t. $n=2a$

Case 2.

$$\begin{aligned} n^2-5n+2 &= (2a)^2-5(2a)+2 \\ &= 4a^2-10a+2 \\ &= 2(2a^2-5a+1) \end{aligned}$$

Therefore n^2-5n+2 is even

Therefore, for any integer n, n^2-5n+2 is even. ∎

Flow:

n any integer $\to$ n is odd $\to$ n^2-5n+2 even

n any integer $\to$ n is even $\to$ n^2-5n+2 even

Scratchwork:

If $7n+4$ is even, there's some $a \in \mathbb{Z}$, $7n+4=2a$

$$7n+4=2a$$

$$7n=2a-4$$

$$n=\frac{2a-4}{7}$$

$$3n-11=\underbrace{\frac{3}{7}}_{\text{fraction!}}(2a-4)-11$$

how to show odd?

n	7n+4		3n-11	
1	11	O	-8	E
2	18	E	-5	O
3	25	O	-2	E
4	32	E	1	O
5	39	O	4	E
6	46	E	7	O

$$\left(\begin{matrix}7n+4\\ \text{even}\end{matrix} \Rightarrow \begin{matrix}3n-11\\ \text{odd}\end{matrix}\right)$$

$$\equiv$$

$$\neg\left(\begin{matrix}7n+4\\ \text{even}\end{matrix}\right) \vee \left(\begin{matrix}3n-11\\ \text{odd}\end{matrix}\right)$$

$$\equiv$$

$$\left(\begin{matrix}7n+4\\ \text{odd}\end{matrix}\right) \vee \left(\begin{matrix}3n-11\\ \text{odd}\end{matrix}\right)$$

If n is odd then $7n+4$ is odd

If n is even then $3n-11$ is odd.

$$\begin{matrix}7n+4\\ \text{even}\end{matrix} \Rightarrow \begin{matrix}n \text{ is}\\ \text{even}\end{matrix} \Rightarrow \begin{matrix}3n-11\\ \text{odd}\end{matrix}$$

(Exercise)

<u>Proposition</u>: Let n be an integer. If $7n+4$ is even then $3n-11$ is odd.

<u>Proof</u>: ~~The statement~~ Let $P(n)$: $7n+4$ is even

$Q(n)$: $3n-11$ is odd.

The statement $P(n) \Rightarrow Q(n)$ is equivalent to

$$\sim P(n) \vee Q(n)$$

so we prove this instead. I.e, we prove

$7n+4$ is odd or $3n-11$ is odd

n is either even or odd: we consider each case separately.

<u>Case 1</u>: n is even. Let $n=2a$. Then

$3n-11 = 6a-11 = 2(3a-6)+1$ is odd.

So ($7n+4$ is odd or $3n-11$ is odd).

<u>Case 2</u>: n is odd. Let $n=2a+1$. Then

$7n+4 = 7(2a+1)+4 = 14a+11 = 2(7a+5)+1$ is odd

So ($7n+4$ is odd or $3n-11$ is odd).

In either case, ($7n+4$ is odd or $3n-11$ is odd). ∎

(Exercise)

Proposition: Let n be an integer. If n is odd, then $8 \mid n^2-1$.

Proof: Since n is odd, there is some $a \in \mathbb{Z}$ such that $n=2a+1$.

Then
$$\begin{aligned} n^2-1 &= (2a+1)^2-1 \\ &= 4a^2+4a \\ &= 4(a^2+a) \end{aligned}$$

Either a is even or a is odd.

Case 1: a is even. There is some $b \in \mathbb{Z}$ s.t. $a=2b$
$$\begin{aligned} 4(a^2+a) &= 4((2b)^2+2b) \\ &= 4(4b^2+2b) \\ &= 8(2b^2+b) \end{aligned}$$
which is a multiple of 8.

Case 2: a is odd. There is some $b \in \mathbb{Z}$ s.t. $a=2b+1$
$$\begin{aligned} 4(a^2+a) &= 4((2b+1)^2+(2b+1)) \\ &= 4(4b^2+6b+2) \\ &= 8(2b^2+3b+1) \end{aligned}$$
which is a multiple of 8.

In either case, $4(a^2+a)$ is a multiple of 8, so n^2-1 is too. $\blacksquare$